Shewhart Control Charts
X-Bar and S Chart: Formula

X-Bar ($\overline{X}$) and S Chart Formula

Data

Month	Cases (n)	Mean time taken to complete assessment ($\overline{X}$)	$n\overline{X}$	S	n S	C$_4$	X-Bar Chart			S Chart			
							A$_3$	LCL	UCL	B$_3$	B$_4$	LCL	UCL
Jan-14	40	3.75	150	1.10	44.14	0.99361	0.477382	3.34	4.59	0.658139	1.341861	0.86	1.76
Feb-14	31	4.58	142	1.12	34.68	0.99170	0.543306	3.25	4.68	0.609474	1.390526	0.80	1.82
Mar-14	34	3.62	123	1.13	38.37	0.99245	0.518393	3.29	4.64	0.627928	1.372072	0.82	1.80
Apr-14	34	4.41	150	1.31	44.38	0.99245	0.518393	3.29	4.64	0.627928	1.372072	0.82	1.80
May-14	24	3.54	85	1.18	28.29	0.98919	0.619000	3.15	4.78	0.555000	1.455000	0.73	1.91
Jun-14	34	4.18	142	1.40	47.70	0.99245	0.518393	3.29	4.64	0.627928	1.372072	0.82	1.80
Jul-14	34	3.79	129	1.68	57.25	0.99245	0.518393	3.29	4.64	0.627928	1.372072	0.82	1.80
Aug-14	28	3.96	111	1.20	33.64	0.99079	0.572196	3.22	4.72	0.587972	1.412028	0.77	1.85
Sep-14	34	3.62	123	1.13	38.37	0.99245	0.518393	3.29	4.64	0.627928	1.372072	0.82	1.80
Oct-14	31	4.06	126	1.18	36.62	0.99170	0.543306	3.25	4.68	0.609474	1.390526	0.80	1.82
Nov-14	31	3.68	114	1.22	37.87	0.99170	0.543306	3.25	4.68	0.609474	1.390526	0.80	1.82
Dec-14	22	3.50	77	1.34	29.40	0.98817	0.647000	3.12	4.81	0.534000	1.466000	0.70	1.92
Jan-15	28	3.96	111	1.71	47.88	0.99079	0.572196	3.22	4.72	0.587972	1.412028	0.77	1.85
Feb-15	26	4.58	119	1.65	42.99	0.99005	0.594232	3.19	4.74	0.571493	1.428507	0.75	1.87
Mar-15	34	4.18	142	1.40	47.70	0.99245	0.518393	3.29	4.64	0.627928	1.372072	0.82	1.80
Total $\sum n_i$	465	Total $\sum(n_i\overline{X}_i)$	1844	Total $\sum(n_i S_i)$	609.29								

X = an individual measurement within a subgroup

n = subgroup size (# of measurements per subgroup)

$\overline{X}$ = subgroup mean = $\sum X / n_i$

S = subgroup standard deviation = $\sqrt{\frac{\sum(X - \overline{X})^2}{n_i - 1}}$

$\overline{\overline{X}}$ = weighted mean of the subgroup means

$\overline{S}$ = weighted mean of the subgroup standard deviations

$\overline{\overline{X}}$	3.97

$\overline{S}$	1.31

Calculations

Start by calculating the weighted mean ($\overline{\overline{X}}$) and weighted standard deviation ($\overline{S}$). These will also be used as the CL for your X-Bar Chart ($\overline{\overline{X}}$) and S Chart ($\overline{S}$) and when calculating the LCLs and UCLs

$$\overline{\overline{X}} = \frac{\sum(n_i\overline{X}_i)}{\sum n_i}$$

$$= \frac{1844}{465}$$

$$= 3.97 \text{ (2 d.p.)}$$

$$\overline{S} = \frac{\sum(n_i S_i)}{\sum n_i}$$

$$= \frac{609.29}{465}$$

$$= 1.31 \text{ (2 d.p.)}$$

SPC uses 3 constants (A$_3$, B$_3$, B$_4$) when calculating the LCLs and UCLs for X-Bar and S Charts. The full list of constants can be found on page 4. Where the subgroup size (n) is greater than 25 (n >25), the formulas below can be used to calculate the constants. These utilise a fourth constant (C$_4$)

$$C_4 = \sqrt{\frac{2}{n-1}} \times \frac{(\frac{n}{2}-1)!}{(\frac{n-1}{2}-1)!}$$

$$A_3 = \frac{3}{C_4\sqrt{n}}$$

$$B_3 = 1 - \frac{3\sqrt{1-(C_4)^2}}{C_4}$$

$$B_4 = 1 + \frac{3\sqrt{1-(C_4)^2}}{C_4}$$

Next, you will have to individually calculate the UCLs and LCLs for each data point ($\overline{X}$) on both the X-Bar and S Charts. This is because subgroup size (n) differs for each data point. The limits for the first subgroup (Jan-14) are calculated below.

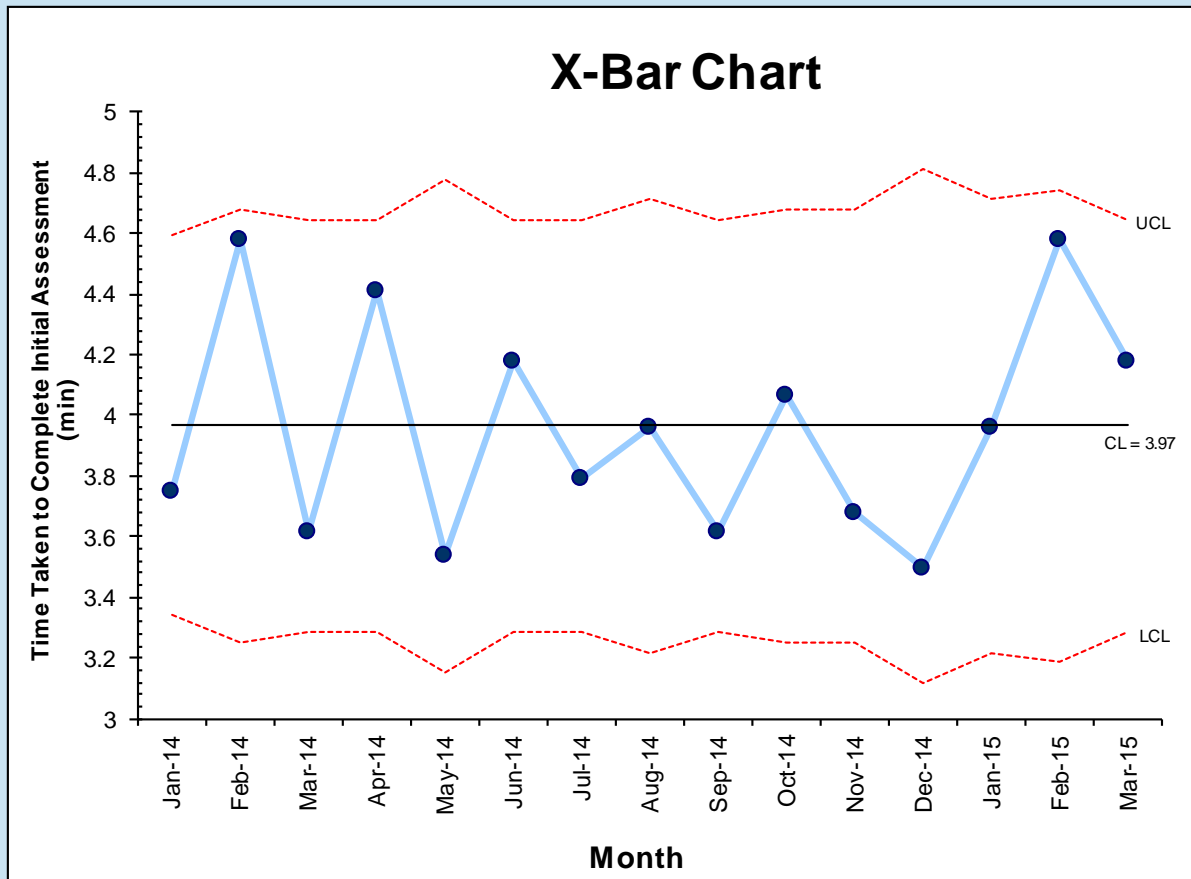| X-Bar Chart	S Chart
$LCL = \overline{\overline{X}} - (A_3 \times \overline{S})$	$LCL = B_3 \times \overline{S}$
$= 3.97 - (0.477382 \times 1.31)$	$= 0.658139 \times 1.31$
$= 3.34 \text{ (2 d.p.)}$	$= 0.86 \text{ (2 d.p.)}$
$UCL = \overline{\overline{X}} + (A_3 \times \overline{S})$	$UCL = B_4 \times \overline{S}$
$= 3.97 + (0.477382 \times 1.31)$	$= 1.341861 \times 1.31$
$= 4.59 \text{ (2 d.p.)}$	$= 1.76 \text{ (2 d.p.)}$

X-Bar ($\overline{X}$) and S Chart Formula

X Bar Chart

This chart helps you understand the changes in performance of the system over time. In this example we are plotting the mean time taken to complete initial assessment ($\overline{X}$) each month.

$$CL = \text{center line } (\overline{\overline{X}})$$

$$LCL = \text{lower control limit } (\overline{\overline{X}} - (A_3 \times \overline{S}))$$

$$UCL = \text{upper control limit } (\overline{\overline{X}} + (A_3 \times \overline{S}))$$
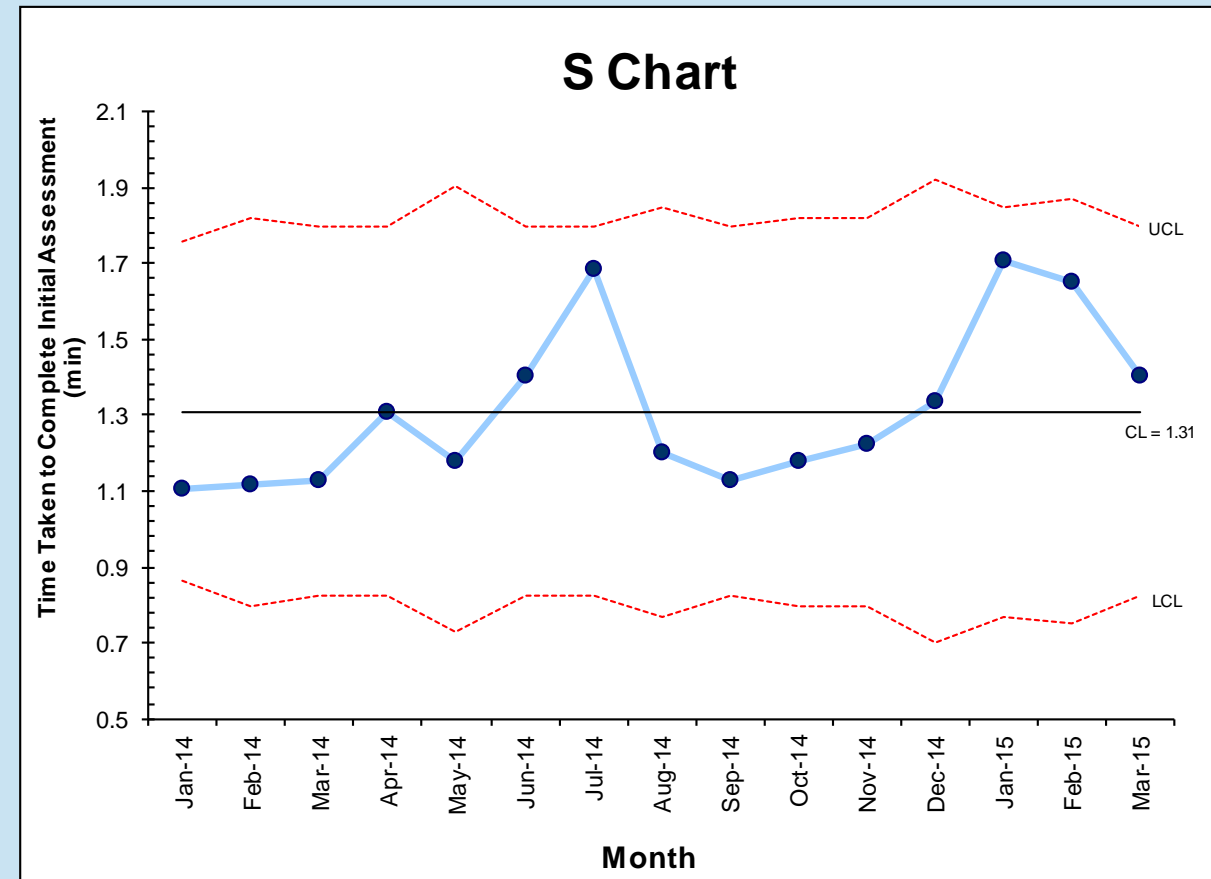
S Chart

This chart helps you to understand the changes in the variation of performance over time. In this example we are plotting the standard deviation of the times taken to complete initial assessment **(S)** each month

$$CL = \text{center line } (\overline{S})$$

$$LCL = \text{lower control limit } (B_3 \times \overline{S})$$

$$UCL = \text{upper control limit } (B_3 \times \overline{S})$$

X-Bar Chart — Time Taken to Complete Initial Assessment (min) vs Month (Jan-14 to Mar-15). CL = 3.97, with UCL and LCL control limits.

S Chart — Time Taken to Complete Initial Assessment (min) vs Month (Jan-14 to Mar-15). CL = 1.31, with UCL and LCL control limits.

X-Bar ($\overline{\text{X}}$) and S Chart Formula

Table of Constants

Used in calculating the LCL and UCL of the X-Bar and S Charts.
For subgroup sizes larger than 25, the constants can be calculated.
All formulas can be found on page 2

Subgroup Size (n)	C_4	A_3	B_3	B_4
2	0.7979	2.6590	0.0000	3.2670
3	0.8862	1.9540	0.0000	2.5680
4	0.9213	1.6280	0.0000	2.2660
5	0.9400	1.4270	0.0000	2.0890
6	0.9515	1.2870	0.0300	1.9700
7	0.9594	1.1820	0.1180	1.8820
8	0.9650	1.0990	0.1850	1.8150
9	0.9693	1.0320	0.2390	1.7610
10	0.9727	0.9750	0.2840	1.7160
11	0.9754	0.9270	0.3210	1.6790
12	0.9776	0.8860	0.3540	1.6460
13	0.9794	0.8500	0.3820	1.6180
14	0.9810	0.8170	0.4060	1.5940
15	0.9823	0.7890	0.4280	1.5720
16	0.9835	0.7630	0.4480	1.5520
17	0.9845	0.7390	0.4660	1.5340
18	0.9854	0.7180	0.4820	1.5180
19	0.9862	0.6980	0.4970	1.5030
20	0.9869	0.6800	0.5100	1.4900
21	0.9876	0.6630	0.5230	1.4770
22	0.9882	0.6470	0.5340	1.4660
23	0.9887	0.6330	0.5450	1.4550
24	0.9892	0.6190	0.5550	1.4550
25	0.9896	0.6060	0.5650	1.4350